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Determination of hardness to elastic modulus ratios using Knoop indentation measurements and a model based on loading and reloading half-cycles

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A model is developed which results in an expression relating the residual in-surface dimension of the minor diagonal of a Knoop indenter through a "residual-width parameter" $(b_R/b^*)^2$ to the hardness-to-modulus ratio (H/E) of elastic–plastic materials. The relationship is shown to predict within reasonable accuracy both the intercept and slope of a plot of $(b_R/b^*)^2$ against (H/E), using data for a variety of materials. The work supplements concepts presented in two previous investigations.

1. Introduction

Measurement of the hardness-to-modulus ratio H/E of elastic–plastic materials is an important prerequisite to the determination of fracture toughness by indentation methods [1]. Two indentation techniques have been developed to date to measure this important parameter. The first, developed by Lawn and Howes [2], demonstrated that H/E could be related to the elastic depth recovery of Vickers indentations. The model adopted was based on maintaining compatibility at the maximum penetration depth for an elastic–plastic loading half-cycle and an elastic unloading or reloading half-cycle. The second technique, developed by Marshall *et al.* [3], related H/E to the residual in-surface dimension of the minor diagonal of a Knoop indentation. This method was based on a model utilizing the superposition of solutions for an elliptical hole subjected to uniaxial stress and the fact that the in-surface dimension of the major diagonal undergoes relatively little length change during unloading. Both techniques were shown to give accurate predictions of H/E through the results of indentation experiments conducted on selected materials, mainly ceramics.

This paper applies the methodology of the Lawn and Howes model [2] to a Knoop indentation and offers a supplemental approach to that adopted by Marshall *et al.* [3]. The model is based on an elastic–plastic loading half-cycle and an elastic unloading or reloading half-cycle, and H/E is related to the residual in-surface dimension of the minor diagonal of a Knoop indentation.

2. Model

Referring to Fig. 1, during the loading half-cycle the material immediately surrounding the indenter undergoes elastic–plastic deformation resulting in the characteristic in-surface contact dimensions $2a^*$ and $2b^*$ at maximum indentation load, P^*. As unloading occurs, elastic recovery takes place along the in-surface dimension of the minor diagonal resulting in a residual dimension $2b_R$. Relatively little change takes place in the in-surface dimension of the major diagonal during unloading.

During indentation, the projected area is $A_p = 2ab$ and the mean contact pressure is $p_0 = P/2ab$ with P being the contact load. Adopting the notation of Lawn and Howes [2] with $a = 7.11b$ and $p_0 = H$ during the loading half-cycle,

$$P = 14.22Hb_H^2 \tag{1}$$

In deriving this expression, "pile-up" or "sink-in" of the plastically-deformed material surrounding the contact zone has been neglected. As can be seen from Equation 1, the indentation mechanics is controlled by the hardness H during the elastic–plastic loading half-cycle.

During the elastic unloading or reloading half-cycle, the mean contact pressure associated with a rigid wedge or cone is adopted as $p_m = E \times \cot \gamma/2(1 - \nu^2)$ [4]. In this expression, E and ν are the modulus of elasticity and Poisson's ratio of the indented material, respectively, and γ is the average half-angle of a Knoop indenter $(\alpha + \beta)/2$. Elastic loading of a half-space may then be generally described as

$$P_E = \left[\left(\frac{14.22E}{2(1 - \nu^2)}\right) \cot \gamma\right] b_E^2 \tag{2}$$

where $2b_E$ is the in-surface dimension of the minor diagonal at any given load P_E.

Referring to Fig. 2, the loading half-cycle is shown as OC and is described by Equation 1. The unloading half-cycle is shown as CD, resulting in a residual

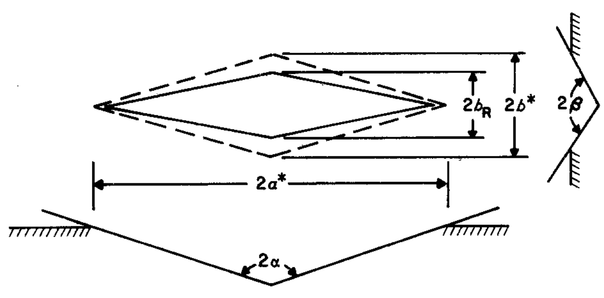

Figure 1 Knoop indentation impression showing in-surface elastic recovery of minor diagonal.

in-surface minor diagonal dimension $2b_R$. Since the unloading and reloading stages are reversible, reloading along path D′C′ will be used to describe the unloading half-cycle. Reloading can be described by writing Equation 2 referenced with respect to a coordinate system with origin at D′ as

$$P' = \left[\left(\frac{14.22E}{2(1-\nu^2)}\right)\cot\gamma\right](b_E^2 - b_R^2) \quad (3)$$

where $b_R \leqslant b_E \leqslant b^*$ during reloading and $2b^*$ is the in-surface dimension of the minor diagonal at maximum load P^*. This is equivalent to adopting the "compressed spring" model of Lawn and Howes [2]. Compatibility conditions at the end of the loading and reloading half-cycles require that at $P = P^*$, $b_H = b_E = b^*$ so that Equations 1 and 3 give

$$\left(\frac{b_R}{b^*}\right)^2 = 1 - [2(1-\nu^2)\tan\gamma]\left(\frac{H}{E}\right) \quad (4)$$

This expression can, of course, be written in terms of the in-surface major diagonal dimension at maximum load $2a^*$ since $a^* = 7.11b^*$. In conducting actual tests, this may be the more convenient formulation since the length of the in-surface major diagonal undergoes relatively little dimensional change on unloading. In either case, Equation 4 indicates that H/E can be directly related to the residual in-surface minor diagonal dimension $2b_R$.

3. Results and discussion

The relationship between the hardness-to-modulus ratio H/E and the residual in-surface minor diagonal dimension $2b_R$ as expressed in Equation 4 was verified by plotting existing data for a variety of materials as $(b_R/b^*)^2$ against H/E in Fig. 3. The data were obtained from Table I and Fig. 2 of Marshall *et al.* [3] and are shown in Table I of this paper. Values of (b_R/a') graphically shown by Marshall *et al.* [3] were converted to the "residual-width parameter" $(b_R/b^*)^2$ by assuming that $a' \cong a^* = 7.11b^*$. The solid line through the data is a least-squares curve fit with an intercept of 1.03 and a slope of -6.19. The intercept and slope predicted from Equation 4, assuming $\nu = 0.25$ and $\gamma = 75°$ (the average half-angle of a Knoop indenter) are 1.00 and -6.99 respectively. These are reasonable results when considering the sensitivity of the quantity $2(1 - \nu^2)\tan\gamma$ to changes in the average half-angle of the indenter γ, as well as the data scatter. The data do not span the elastic–plastic spectrum but ideal plastic behaviour is indicated in Fig. 3 at the point $(b_R/b^*)^2 = 1.00$, $H/E = 0$. Extrapolation of the curve fit to ideal elastic behaviour gives $(b_R/b^*)^2 = 0$, $H/E = 0.17$.

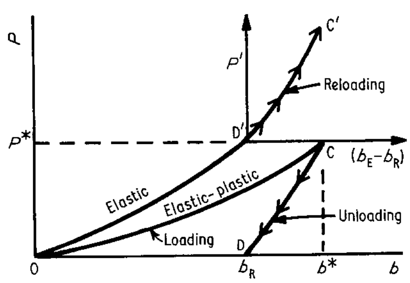

Figure 2 Loading, unloading and reloading half-cycles. P is the contact load and b the minor diagonal in-surface half dimension. Loading plot from Equation 1; reloading plot from Equation 2.

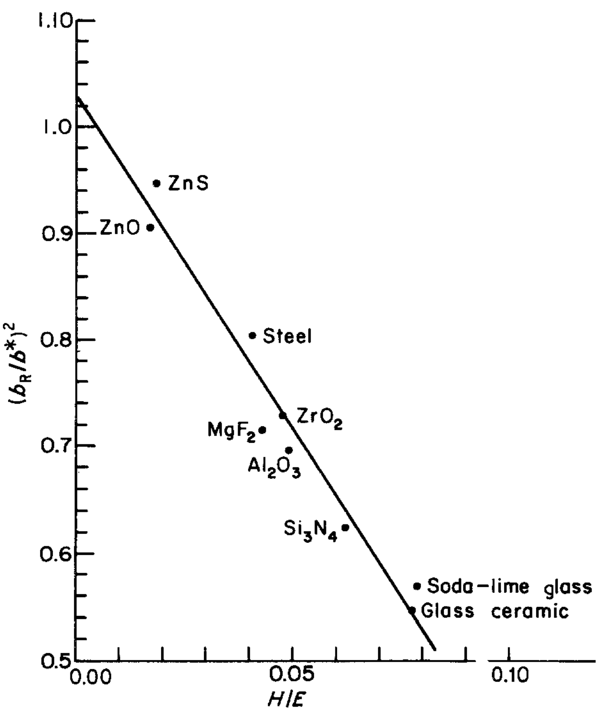

Figure 3 Residual width parameter $(b_R/b^*)^2$ as a function of hardness-to-modulus ratio (H/E).

Results obtained by using the model presented in this paper compare well with those obtained by Marshall *et al.* [3], as can be seen by comparing like intercepts and slopes of plots for a residual-width parameter against hardness-to-modulus ratio for like data. Converting the ordinate of Fig. 3 of this work from $(b_R/b^*)^2$ to $b_R/a^* = b'/a'$ for $b^* = 7.11a^*$ gives intercept of 0.14 and a slope of -0.48, which compare

TABLE I Material and dimensional parameters

Material	H/E	b_R/a^*	$(b_R/b^*)^2$
Soda-lime glass	0.079	0.106	0.568
Glass–ceramic	0.078	0.104	0.547
Si_3N_4 (hot-pressed)	0.062	0.111	0.623
Al_2O_3 (hot-pressed)	0.050	0.118	0.704
ZrO_2 (partially stabilized)	0.048	0.120	0.728
MgF_2 (hot-pressed)	0.043	0.119	0.716
Steel (hardened)	0.041	0.126	0.803
ZnS (hot-pressed)	0.019	0.137	0.949
ZnO (hot-pressed)	0.017	0.134	0.908

favourably with values of 0.14 and −0.45 obtained by Marshall *et al.* [3]. The model described in this paper, however, avoids the assumption of a two-dimensional elliptical hole solution and allows direct analytical determination of the slope and intercept with reasonable accuracy.

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